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MEMORANDUM / MARKING GUIDELINE

GRADE 11 · MATHEMATICS · 2026

TRIGONOMETRY: PROBLEMS IN TWO DIMENSIONS

TOTAL: 50 MARKS

TIME: 1 HOUR

EXAMINER: MR SIMONS

Each ✓ = one mark. Accept any correct alternative method. Accept answers that differ in the last decimal place through rounding, provided the method is correct.

QUESTION 1

1.1	$b^2 = a^2 + c^2 - 2ac \cdot \cos \hat{B}$ ✓✓	✓✓ the cosine rule stated correctly	(2)
1.2	$AC^2 = AB^2 + BC^2 - 2(AB)(BC) \cos \hat{A}BC$ ✓ $AC^2 = 45^2 + 62^2 - 2(45)(62) \cos 105^\circ$ ✓ $\therefore AC = 85,52 \text{ m}$ ✓	✓ the cosine rule ✓ correct substitution ✓ answer	(3)
1.3	$\text{Area } \triangle ABC = \frac{1}{2}(45)(62) \sin 105^\circ$ ✓ $= 1\,347,47 \text{ m}^2$ ✓	✓ correct substitution ✓ answer	(2)
1.4	$AD^2 = AC^2 + CD^2 - 2(AC)(CD) \cos \hat{A}CD$ ✓ $AD^2 = 85,52^2 + 58^2 - 2(85,52)(58) \cos 48^\circ$ ✓ $\therefore AD = 63,56 \text{ m}$ ✓	✓ the cosine rule ✓ correct substitution ✓ answer	(3)
1.5	$\text{Area } \triangle ACD = \frac{1}{2}(85,52)(58) \sin 48^\circ$ $= 1\,843,00 \text{ m}^2$ ✓✓ $\text{Area } ABCD = \text{Area } \triangle ABC + \text{Area } \triangle ACD$ $= 1\,347,47 + 1\,843,00 = 3\,190,47 \text{ m}^2$ ✓	✓✓ area of $\triangle ACD$ ✓ answer	(3)
		TOTAL	[13]

QUESTION 2

$$h = \frac{AB \cdot \sin \alpha \cdot \sin \beta}{\sin(\beta - \alpha)}$$

2.1	$\hat{A}TB = \beta - \alpha$ ✓ (<i>exterior angle of $\triangle ABT$</i>)	✓ $\beta - \alpha$, exterior angle	(1)
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2.2	$\hat{A}BT = 180^\circ - \beta \quad \checkmark \quad (\text{angles on a straight line})$ $\hat{A}TB = \beta - \alpha \quad \checkmark \quad (\text{angle sum of } \triangle ABT)$ In $\triangle ABT$, by the sine rule: $\frac{BT}{\sin \alpha} = \frac{AB}{\sin(\beta - \alpha)} \quad \checkmark$ $\therefore BT = \frac{AB \sin \alpha}{\sin(\beta - \alpha)} \quad \checkmark$ In $\triangle BFT$, $\hat{B}FT = 90^\circ$, so $h = TF = BT \sin \beta \quad \checkmark$ $\therefore h = \frac{AB \cdot \sin \alpha \cdot \sin \beta}{\sin(\beta - \alpha)}$	$\checkmark \hat{A}BT = 180^\circ - \beta$ $\checkmark \hat{A}TB = \beta - \alpha$ $\checkmark$ the sine rule $\checkmark$ BT in terms of AB $\checkmark h = BT \sin \beta$	(5)
2.3	$h = \frac{60 \cdot \sin 28^\circ \cdot \sin 47^\circ}{\sin(47^\circ - 28^\circ)} \quad \checkmark \checkmark$ $= \mathbf{63,28 \text{ m}} \quad \checkmark$	$\checkmark \checkmark$ correct substitution $\checkmark$ answer	(3)
2.4	$BT = \frac{60 \sin 28^\circ}{\sin 19^\circ} \quad \checkmark$ $= \mathbf{86,52 \text{ m}} \quad \checkmark$	$\checkmark$ correct substitution $\checkmark$ answer	(2)
2.5	$\hat{A}BT = 180^\circ - 47^\circ = 133^\circ \quad \checkmark$ Area $\triangle ABT = \frac{1}{2}(60)(86,52) \sin 133^\circ \quad \checkmark$ $= \mathbf{1\,898,31 \text{ m}^2} \quad \checkmark$	$\checkmark \hat{A}BT = 133^\circ$ $\checkmark$ correct substitution $\checkmark$ answer	(3)
		TOTAL	[14]

QUESTION 3

$$\cos \theta = \frac{16 - 3x^2}{8x}$$

3.1	$PR^2 = PQ^2 + QR^2 - 2(PQ)(QR) \cos \theta \quad \checkmark$ $(2x)^2 = 4^2 + x^2 - 2(4)(x) \cos \theta \quad \checkmark$ $4x^2 = 16 + x^2 - 8x \cos \theta$ $8x \cos \theta = 16 + x^2 - 4x^2 = 16 - 3x^2 \quad \checkmark$ $\therefore \cos \theta = \frac{16 - 3x^2}{8x} \quad \checkmark$	$\checkmark$ the cosine rule $\checkmark$ correct substitution $\checkmark$ simplification $\checkmark$ answer	(4)
3.2	$\cos \theta = \frac{16 - 27}{24} = \frac{-11}{24} \quad \checkmark$ $\therefore \theta = \mathbf{117,28^\circ} \quad \checkmark$	$\checkmark$ correct substitution $\checkmark$ answer	(2)
3.3	Area $\triangle PQR = \frac{1}{2}(PQ)(QR) \sin \theta \quad \checkmark$ $= \frac{1}{2}(4)(3) \sin 117,28^\circ \quad \checkmark$ $= \mathbf{5,33 \text{ square units}} \quad \checkmark$	$\checkmark$ the area rule $\checkmark$ correct substitution $\checkmark$ answer	(3)

3.4	<i>A triangle needs $0^\circ < \theta < 180^\circ$, so $-1 < \cos \theta < 1$.</i> <i>Note $x > 0$, since x and $2x$ are lengths.</i> $\cos \theta < 1$: $16 - 3x^2 < 8x \quad \therefore 3x^2 + 8x - 16 > 0$ $(3x - 4)(x + 4) > 0$ <i>critical values $x = \frac{4}{3}$ and $x = -4$</i> <i>opens upwards,</i> <i>so positive OUTSIDE the critical values:</i> $x < -4$ or $x > \frac{4}{3}$ <i>but $x > 0$, so $x > \frac{4}{3}$ ✓</i> $\cos \theta > -1$: $16 - 3x^2 > -8x \quad \therefore 3x^2 - 8x - 16 < 0$ $(3x + 4)(x - 4) < 0$ <i>critical values $x = -\frac{4}{3}$ and $x = 4$</i> <i>negative BETWEEN the critical values:</i> $-\frac{4}{3} < x < 4$ <i>but $x > 0$, so $x < 4$ ✓</i> $\therefore \frac{4}{3} < x < 4$ ✓ <i>[Accept the triangle inequality on the sides 4, x and $2x$ for FULL marks: $4 + x > 2x$ gives $x < 4$ ✓, and $x + 2x > 4$ gives $x > 1\frac{1}{3}$ ✓, hence the same interval ✓. The third case, $4 + 2x > x$, holds for every $x > 0$.]</i> <i>[Why the inequality is STRICT at both ends. At $x = 1\frac{1}{3}$: $\cos \theta = 1$, so $\theta = 0^\circ$, and $QR + PR = 1\frac{1}{3} + 2\frac{2}{3} = 4 = PQ$, so R lands ON the segment PQ. At $x = 4$: $\cos \theta = -1$, so $\theta = 180^\circ$, and $PQ + QR = 4 + 4 = 8 = PR$, so Q lands ON the segment PR. Each endpoint is the triangle inequality at EQUALITY, and the triangle collapses to a straight line. Do not accept $\leq$.]</i>	✓ lower bound $x > 1\frac{1}{3}$ ✓ upper bound $x < 4$ ✓ the interval	(3)
		TOTAL	[12]

QUESTION 4

4.1	$\hat{A}CB = 180^\circ - 34^\circ - 47^\circ = 99^\circ$ ✓	✓ answer	(1)
4.2	$\frac{AC}{\sin \hat{A}BC} = \frac{AB}{\sin \hat{A}CB}$ ✓ $AC = \frac{8,4 \sin 47^\circ}{\sin 99^\circ}$ ✓ $= 6,22 \text{ m}$ ✓	✓ the sine rule ✓ correct substitution ✓ answer	(3)

4.3	$h = AC \sin 34^\circ = 6,22 \sin 34^\circ \quad \checkmark$ $= \mathbf{3,48 \text{ m}} \quad \checkmark$	$\checkmark$ correct substitution $\checkmark$ answer	(2)
4.4	$\text{Area} = \frac{1}{2}(AB)(h) = \frac{1}{2}(8,4)(3,48) \quad \checkmark$ $= \mathbf{14,61 \text{ m}^2} \quad \checkmark$ <i>[Accept 14,62 m² from the rounded height, or the area rule $\frac{1}{2}(AC)(BC) \sin 99^\circ$.]</i>	$\checkmark$ correct substitution $\checkmark$ answer	(2)
4.5	$AD = \frac{1}{2}(8,4) = 4,2 \text{ m}$ In $\triangle ACD$: $CD^2 = AC^2 + AD^2 - 2(AC)(AD) \cos 34^\circ \quad \checkmark$ $CD^2 = 6,22^2 + 4,2^2 - 2(6,22)(4,2) \cos 34^\circ \quad \checkmark$ $\therefore CD = \mathbf{3,61 \text{ m}} \quad \checkmark$	$\checkmark$ the cosine rule $\checkmark$ correct substitution $\checkmark$ answer	(3)
		TOTAL	[11]

TOTAL: 50 MARKS

COGNITIVE LEVEL ANALYSIS

CAPS weighting is written for a full Paper 2. A single-topic application test cannot carry 20% recall, so Levels 1 and 2 are reported against the target rather than forced onto it; higher order (Levels 3 and 4) is the figure that is held to a minimum of 40%.

Cognitive level	Marks	%	Target	Questions
Level 1: Knowledge	4	8%	20%	1.1, 2.1, 4.1
Level 2: Routine procedures	25	50%	35%	1.2, 1.3, 1.4, 2.3, 2.4, 3.2, 3.3, 4.2, 4.3, 4.4
Level 3: Complex procedures	13	26%	30%	1.5, 2.5, 3.1, 4.5
Level 4: Problem solving	8	16%	15%	2.2, 3.4
Higher order (Levels 3 + 4)	21	42%	40% min	
TOTAL	50	100%	100%	